Research Note

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No Pot of Gold at the End of Program Spectrum Rainbow: Greatest Risk Evaluation Formula Does Not Exist

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Abstract

Spectrum Based Fault Localisation (SBFL) techniques rely on risk assessment formulæ to convert program execution spectrum into risk evaluation values, which are in turn used to rank program statements according to their relative suspiciousness with respect to the observed failure. Recent work proved equivalence and hierarchy between different formulæ, identifying a few groups of *maximal* formulæ, i.e., formulæ that do not dominate each other. The holy grail in the field has been to come up with the greatest formula, that is, the one that dominates all known formulæ. This paper proves that such a formula does not exist.

1 Introduction

Spectrum Based Fault Localisation (SBFL) is an automated debugging aid technique that assigns risk evaluation values to program statements. The values are calculated from the spectrum data, obtained during test execution. The most widely adopted form of the spectrum data is in the form of a tuple $\sigma = (e_p, e_f, n_p, n_f)$, collected per program statement: e_p and e_f represent the number of tests that execute the statement and pass and fail respectively, while n_p and n_f represent the number of tests that do not execute the statement and pass and fail respectively. A risk assessment formula R is a formula that converts the spectrum data into the risk evaluation value, based on which program statements are either ranked for humans [14] or assigned probabilities of being mutated for the purpose of automated program repair [16].

A large part of the literature focuses on designing and empirically evaluation different formulæ [1, 3, 9, 17]. More recently, Genetic Programming (GP) has been applied to automatically evolve risk evaluation formulæ [20].

Empirical evaluation of formulæ using known faults has been the de facto standard way of comparing performance of different formulæ. However, Recent work showed that theoretical comparison of formulæ is possible [11] and proved equivalence and hierarchy between formulæ groups [18].

A formula R_1 is *better* than another formula R_2 when it can be proved that R_1 *always* ranks the faulty statement higher than or equal to R_2, *regardless* of the program, the test suite, and the fault. The proved hierarchy between formulæ can be summarised as a forest of directed trees [18]. There are multiple hierarchical trees, at the tops of which exist non-dominated, *maximal* groups. Formulæ from different maximal groups do not dominate each other. That is, R_1 outperforms R_2 for certain combinations of program, test suites, and faults, whereas R_2 outperforms R_1 for others.

Subsequently, the formulæ evolved by GP has been proved to be equivalent to the best known formulæ designed by human [19].

While the theoretical framework provided an alternative to empirical evaluation of different formulæ, there were limitations. First, the hierarchy was only proved within the set of 50 studied formulæ, and not the entire set of all possible formulæ. Second, it did not provide any insights into the existence of the *greatest* formula, i.e. one that would outperform all other formulæ.

This paper

This paper proves that the *greatest* formula, i.e. one that is better than all other formulæ, does not exists. The paper also proposes a novel visualisation technique that can present SBFL formulæ in an intuitive way.

2 Background

2.1 Spectrum-Based Fault Localisation

2.1.1 Basic concept

Spectrum-Based Fault Localisation (SBFL) refers to a group of techniques that use program spectra to find the location of the fault in the given program that causes certain tests to fail. Program spectra can be best described as a summary of a set of program executions [5]. For the SBFL techniques, the most widely used type of program spectra is the combination of code coverage and the test results, on which this paper focuses too. Suppose SUT has n statements, and the test suite contains m test cases: the program spectrum for SBFL can be described as a matrix of n rows and 4 columns. Each row corresponds to individual statement of SUT, and contains the tuple (e_f, e_p, n_f, n_p). Members e_f and e_p represent the number of times the corresponding program statement has been executed by tests, with fail and pass as a result respectively. Similarly, n_f and n_p represent the number of times the corresponding program statement has *not* been executed by tests, with fail and pass as a result respectively[1]. SBFL techniques subsequently use a risk evaluation formula, which is a formula based on the four counters, to predict the relative risk of each statement containing the fault. Compared to the case in which the developer investigates the structural elements in the order from s_1 to s_9, the ranking according to Tarantula produces 66.66% reduction in debugging effort (i.e. the developer will encounter s_7 6 elements earlier).

Structural Elements	Test t_1	Test t_2	Test t_3	Spectrum e_f	e_p	n_f	n_p	Tarantula	Rank
s_1	•			0	1	2	0	0.00	9
s_2	•			0	1	2	0	0.00	9
s_3	•			0	1	2	0	0.00	9
s_4	•			0	1	2	0	0.00	9
s_5	•			0	1	2	0	0.00	9
s_6	•		•	1	1	1	0	0.33	4
s_7 (faulty)		•	•	2	0	0	1	1.00	1
s_8	•	•		1	1	1	0	0.33	4
s_9	•	•	•	2	1	0	0	0.50	2
Result	P	F	F						

Table 1: Motivating Example: the faulty statement s_7 achieves the 1st place when ranked according to the Tarantula risk evaluation formula in Equation 1.

For example, Table 1 illustrates how the Tarantula metric [7], defined in Equation 1, can be applied to a small exemplar program spectrum. Suppose the structural element s_7 contains the fault. The coverage relationship between structural elements and the given test suite $T = \{t_1, t_2, t_3\}$ is given in the second column, with the corresponding test results. The Spectrum column contains the program spectrum data for T; the column Tarantula contains the resulting risk evaluation metric values. Finally, the column Rank contains the ranking of structural elements according to the Tarantula metric values. The faulty statement, s_7, is assigned with the highest Tarantula metric value, and therefore ends up in the first place.

$$\text{Tarantula} = \frac{\frac{e_f}{e_f+n_f}}{\frac{e_p}{e_p+n_p} + \frac{e_f}{e_f+n_f}} \tag{1}$$

2.1.2 Risk evaluation formula

Based on Section 2.1.1, a SBFL risk evaluation formula is a function from program spectrum to suspiciousness score, such as Tarantula in Equation 1. More formally, it is defined as follow:

Definition 2.1. A risk evaluation formula R is a member of set $\mathbb{R} = \{R | R : I \times I \times I \times I \rightarrow Real\}$ (where I denotes the set of non-negative integers and $Real$ denotes the set of real numbers), which maps $A_i =< e^i_f, e^i_p, n^i_f, n^i_p >$ of each statement s_i to its risk value.

These formulæare either designed by human [9, 11] or by Genetic Programming [20]. Table 2 contains several of the most widely studied formulæ. Interestingly, Jaccard [6] and Ochiai [12] were first studied in Botany and Zoology respectively but have been subsequently studied in the context of fault localisation [2,

[1]The sum of e_f, e_p, n_f, and n_p should be m.

11]. Tarantula was originally developed as a visualisation method [8, 9] but also increasingly considered as an SBFL risk evaluation formula independent from visualisation [7, 13]. AMPLE [4] and three different versions of Wong metric [17] have been introduced specifically for fault localisation. Lately, Genetic Programming (GP) has been applied to SBFL: instead of being manually designed, risk evaluation formulæ were evolved by GP from given fault datasets [20].

Table 2: SBFL formulæ

	Name	Formula expression
ER'_1	Naish1	$\begin{cases} -1 & \text{if } e_f < F \\ n_p & \text{if } e_f = F \end{cases}$
	Naish2	$e_f - \frac{e_p}{e_p+n_p+1}$
	GP13	$e_f(1 + \frac{1}{2e_p+e_f})$
ER_5	Wong1	e_f
	Russel & Rao	$\frac{e_f}{e_f+n_f+e_p+n_p}$
	Binary	$\begin{cases} 0 & \text{if } e_f < F \\ 1 & \text{if } e_f = F \end{cases}$
GP02		$2(e_f + \sqrt{n_p}) + \sqrt{e_p}$
GP03		$\sqrt{\lvert e_f^2 - \sqrt{e_p}\rvert}$
GP19		$e_f\sqrt{\lvert e_p - e_f + n_f - n_p\rvert}$

Naish1 and Naish2 metrics are recent additions to SBFL techniques that showed an interesting research direction: these metrics are designed with accompanying proof that shows they produce optimal ranking, as long as the fault is located in a specific program structure (two consecutive `If-Then-Else` blocks, called `ITE2`) [11]. It was the first attempt at theoretical analysis of SBFL formulæ. Subsequently, Xie et al. [18] proved the hierarchy between formulæ, showing that there exist multiple hierarchy trees. Some formulæ were proven to be equivalent to others (i.e. belong to the same node in the hierarchy tree) with respect to the ranking of the faulty statement. The formulæ at the top are called *maximal* because they dominated others (i.e. always produce higher ranking for the faulty stataement than the other) in the tree. However, no globally maximal formula is known; all the known maximals did not dominate each other. Some of the GP evolved formulæ [20] were proven to be maximal, but not globally maximal.

2.2 Theoretical framework

With the development of more and more risk evaluation formulæ, people began to investigate their performance. The most commonly adopted effectiveness measurement is referred to as Expense metric, which is the percentage of code that needs to be examined before the faulty statement is identified [20]. A lower Expense of formula R indicates a better performance. Recently a theoretical framework [18] has been developed to compare the performances of different formulæ against any combinations of programs, faulty statements, and consistent tie-breaking schemes [2]. The consistent tie-breaking scheme, and the relation-

[2] In practice, a tie-breaking scheme may be required to determine the order of the statements with same risk values.

ships between different formulæ, are defined as follows:

Definition 2.2 (Consistent tie-breaking scheme)**.** Given any two sets of statements S_1 and S_2, which contain elements having the same risk values. A tie-breaking scheme returns the ordered statement lists O_1 and O_2 for S_1 and S_2, respectively. The tie-breaking scheme is said to be consistent, if all elements common to S_1 and S_2 have the same relative order in both O_1 and O_2.

Let R_1 and R_2 be two risk evaluation formulæ in $\mathbb{R}$, and E_1 and E_2 denote the Expenses with respect to the same faulty statement for R_1 and R_2, respectively. We define two types of relations between R_1 and R_2 as follows.

Definition 2.3 (Better)**.** R_1 is said to be *better* than R_2 (denoted as $R_1 \rightarrow R_2$) if, for any program, faulty statement s_f, test suite, and consistent tie-breaking scheme, E_1 is less than or equal to E_2.

Definition 2.4 (Equivalent)**.** R_1 and R_2 are said to be *equivalent* (denoted as $R_1 \leftrightarrow R_2$), if, for any program, faulty statement s_f, test suite and consistent tie-breaking scheme, E_1 is equal to E_2.

It follows from the definition that $R_1 \rightarrow R_2$ means R_2 is not more effective than R_1. As a reminder, if both $R_1 \rightarrow R_2$ and $R_2 \rightarrow R_1$ hold, then it follows that $R_1 \leftrightarrow R_2$; if $R_1 \rightarrow R_2$ holds but $R_2 \rightarrow R_1$ does not hold, $R_1 \rightarrow R_2$ is said to be a strictly "*better*" relation.

In the theoretical framework, there are several assumptions, which are listed as follows:

1. A test oracle exists, that is, for any test case, the testing result of either *fail* or *pass*, can be decided.
2. We exclude two types of faults that SBFL is not designed for. The first type includes omission faults. We assume that for all observed failures, the execution of a faulty statement s_f is the cause. In other words, a statement cannot cause a failure by not being executed. A more generalised statement would be that SBFL techniques cannot localise omission faults (i.e. statements forgotten by the programmer, such as a missing null check). The second type includes the non-deterministic faults. SBFL techniques expect the same test input to produce the same program spectrum with every execution.
3. The fault is executed by the test suite. Being a type of dynamic analysis, SBFL techniques cannot localise faults in statements that are not covered by the test suite.
4. For each fault that needs to be localised, the test suite contains at least one passing test case and one failing test case.

As a reminder, our analysis only focuses on statements that are covered by the given test suite, since only these statements are possible to be the faulty statement that triggers the observed failure. And thus the statements that is never executed by any test case in the given test suite should be ignored or assigned with the lowest risk values. Moreover, our analysis only consider programs with single fault. For readers who are interested in the justifications, validity and impacts of the above assumptions, please refer to the previous work [18].

In order to compare two risk evaluation formulæ in $\mathbb{R}$ under the above definitions of relations, the previous work [18] have provided a theoretical framework, which divides all statements into three mutually exclusive subsets, as follows.

Definition 2.5. Given a program with n statements $PG =< s_1, s_2, ..., s_n >$, a test suite of m test cases $TS = \{t_1, t_2, ..., t_m\}$, and a risk evaluation formula R, which assigns a risk value to each program statement. For each statement s_i, a spectrum vector $\sigma(s_i) =< e^i_f, e^i_p, n^i_f, n^i_p >$ can be constructed from TS, and $R(e^i_f, e^i_p, n^i_f, n^i_p)$ is a risk evaluation formula that assigns a risk value to statement s_i. For any faulty statement s_f, it is possible to define the following three sets of statements:

$$S_B^R = \{s_i \in S | R(e_f^i, e_p^i, n_f^i, n_p^i) > R(e_f^f, e_p^f, n_f^f, n_p^f), 1 \le i \le n\}$$
$$S_F^R = \{s_i \in S | R(e_f^i, e_p^i, n_f^i, n_p^i) = R(e_f^f, e_p^f, n_f^f, n_p^f), 1 \le i \le n\}$$
$$S_A^R = \{s_i \in S | R(e_f^i, e_p^i, n_f^i, n_p^i) < R(e_f^f, e_p^f, n_f^f, n_p^f), 1 \le i \le n\}$$

That is, statements in S_B^R have higher risk values than s_f, and thus are all ranked above any statements in S_F^R; statements in S_F^R have the same equal risk value as that of s_f and, thus, are all ranked in the middle of the ranking list, together with s_f (tie-breaking scheme is needed to further distinguish them); and statements in S_A^R have lower risk values than s_f and, thus, are all ranked below any statements in S_F^R.

In the current framework, two results have been developed for establishing the relationship between two risk evaluation formulæ. They are as follows:

Theorem 2.6. Given any two risk evaluation formulæ R_1 and R_2 from $\mathbb{R}$, if, for any program, faulty statement s_f ,and test suite, it holds that $S_B^{R_1} \subseteq S_B^{R_2} \wedge S_A^{R_2} \subseteq S_A^{R_1}$, then $R_1 \to R_2$.

Theorem 2.7. Let R_1 and R_2 be two risk evaluation formulæ from $\mathbb{R}$. If, for any program, faulty statement s_f, and test suite, it holds that $S_B^{R_1} = S_B^{R_2} \wedge S_F^{R_1} = S_F^{R_2} \wedge S_A^{R_1} = S_A^{R_2}$, then $R_1 \leftrightarrow R_2$.

With the theoretical framework, Xie et al. have investigated 30 formulæ, among which six equivalent groups of formulæ (namely, ER_1 to ER_6) have been identified and two of them are maximal groups of formulæ to the investigated formulæ [18]. The detailed and complete proofs that formulæ within each group share the same set subdivision can be found in the previous work [18]. The definition of limited maximality, i.e. maximality with respect to $\mathbb{S}$, is as follows:

Definition 2.8. Limited Maximality. A risk evaluation formula R_1 from a subset of formulæ, $\mathbb{S} \subset \mathbb{R}$, is said to be a maximal formula of $\mathbb{S}$ if for any element $R_2 \in \mathbb{S}$, $R_2 \to R_1$ implies $R_2 \leftrightarrow R_1$.

3 Maximal and Greatest Formulæ

The existing definition of a maximal formula in Definition 2.8 only concerned a subset of formulæ, $\mathbb{S}$, out of all possible formulæ, $\mathbb{R}$. The subset $\mathbb{S}$ contained only 50 formulæ, 30 manually designed ones and 20 GP-evolved ones. The five identified maximal groups are only with respect to these 50 formulæ. Now, let us generalise our analysis by replacing $\mathbb{S}$ with $\mathbb{R}$. This will, in turn, lead to the investigation of the "greatest" formula.

3.1 Preliminaries

3.1.1 Spectral Coordinate

Let us first present some definitions and lemmas. Given a test suite TS, let T denote its size, F denote the number of *failed* test cases and P denote the number of *passed* test cases. From the definitions and the earlier assumptions, it follows that $1 \le F < T$, $1 \le P < T$, and $P + F = T$, as well as the following lemmas:

Lemma 3.1. For any $\sigma(s_i) =< e_f^i, e_p^i, n_f^i, n_p^i >$, it holds that $e_f^i + e_p^i > 0 \wedge e_f^i + n_f^i = F \wedge e_p^i + n_p^i = P \wedge e_f^i \le F \wedge e_p^i \le P$.

Lemma 3.2. For any faulty statement s_f with $\sigma(s_f) =< e_f^f, e_p^f, n_f^f, n_p^f >$, if s_f is the only faulty statement in the program, it follows that $e_f^f = F \wedge n_f^f = 0$.

For a given pair of program and test suite, the values of F and P are constants. Thus for each statement s_i, it follows that $\sigma(s_i) =< e_f^i, P - n_p^i, F - e_f^i, n_p^i >$ after Lemma 3.1, which can be denoted as $\bar{\sigma}(s_i) =<$

Table 3: Definition of investigated formulas

	Name	Formula as $R(e^i_f, e^i_p, n^i_f, n^i_p)$	$\overline{R}(e^i_f, n^i_p)$ with (P, F)
ER'_1	Naish1	$\begin{cases} -1 & \text{if } e_f < F \\ n_p & \text{if } e_f = F \end{cases}$	$\begin{cases} -1 & \text{if } e_f < F \\ n_p & \text{if } e_f = F \end{cases}$
	Naish2	$e_f - \frac{e_p}{e_p+n_p+1}$	$e_f - \frac{P-n_p}{P+1}$
	GP13	$e_f(1 + \frac{1}{2e_p+e_f})$	$e_f(1 + \frac{1}{2(P-n_p)+e_f})$
ER_5	Wong1	e_f	e_f
	Russel & Rao	$\frac{e_f}{e_f+n_f+e_p+n_p}$	$\frac{e_f}{F+P}$
	Binary	$\begin{cases} 0 & \text{if } e_f < F \\ 1 & \text{if } e_f = F \end{cases}$	$\begin{cases} 0 & \text{if } e_f < F \\ 1 & \text{if } e_f = F \end{cases}$
GP2		$2(e_f + \sqrt{n_p}) + \sqrt{e_p}$	$2(e_f + \sqrt{n_p}) + \sqrt{P - n_p}$
GP3		$\sqrt{\lvert e_f^2 - \sqrt{e_p} \rvert}$	$\sqrt{\lvert e_f^2 - \sqrt{P - n_p} \rvert}$
GP19		$e_f\sqrt{\lvert e_p - e_f + n_f - n_p \rvert}$	$e_f\sqrt{\lvert F + P - 2e_f - 2n_p \rvert}$

$e^i_f, e^i_p >$. That is, program spectrum contains two independent parameters in a specific context (i.e. a pair of a program and a test suite), and not four.

Consequently, it is possible to formulate $\overline{\mathbb{R}} = \{\overline{R} | \overline{R} : I_f \times I_p \rightarrow Real\}$, where I_f denotes the set of integers within $[0, F]$ and I_p denotes the set of integers within $[0, P]$, such that $\overline{R}(e^i_f, n^i_p) = R(e^i_f, e^i_p, n^i_f, n^i_p)$). In the subsequent discussion, when two formulæ from $\mathbb{R}$ are compared, it is assumed that they are being applied to the same program and test suite. Thus, in the context of such comparisons, symbols R and $\overline{R}$ can and will be used interchangeably, as are symbols $\mathbb{R}$ and $\overline{\mathbb{R}}$.

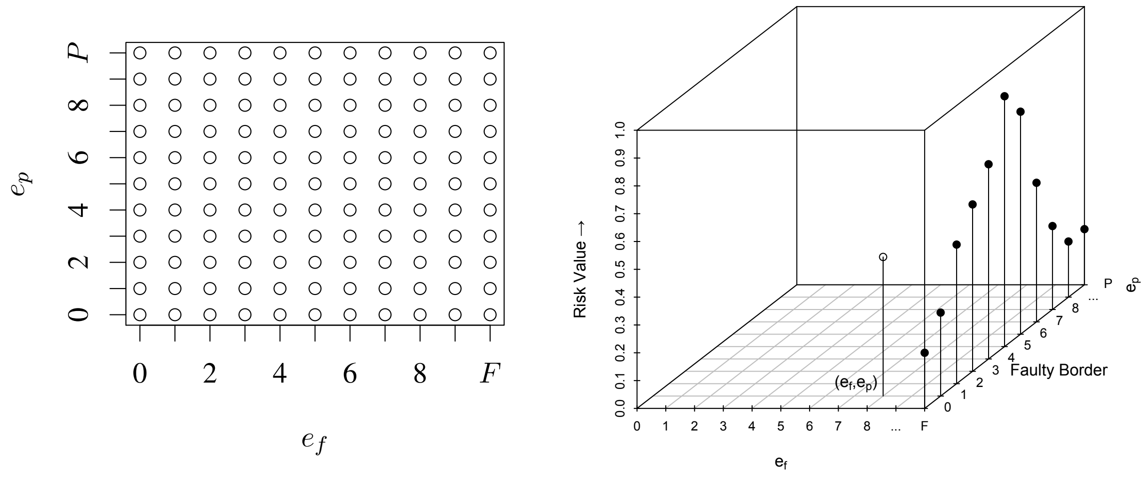
Given any values of P and F, the input domain of any formula $\overline{R}$ is shown as the grid in Figure 1a, where both e_f and e_p are non-negative integers and $0 \leq e^i_f \leq F$ and $0 \leq e^i_p \leq P$. Given a pair of test suite and program, each point (e_f, e_p) on this grid is associated with a group of statements that have the corresponding e_f and e_p values. Note that the number of statements that associated with each point (e_f, e_p) is independent of the formula, but solely decided by the pair of program and test suite.

A formula $\overline{R}$ maps each point (e_f, e_p) to a real number that is the risk value of all statements associated with this point, as shown in Figure 1b. Any assignment of risk values is independent of the number of statements associated with each point (e_f, e_p), but solely decided by the definition of $\overline{R}$.

3.1.2 Analysis of SBFL Space

Lemma 3.2 allows us to limit the region of the input domain $\overline{A}$ in which the faulty statement can be.

Definition 3.3 (Faulty Border). Let us call the sequential points $< (F, 0), (F, 1), ..., (F, e_p), ..., (F, P) >$



(a) Spectral Coordinate $\bar{\sigma}$ for SBFL formulæ (b) Mapping from $\bar{\sigma}$ to risk values by formula $\overline{R}$

Figure 1: Visualising the SBFL Space

$(0 \leq e_p \leq P)$ the *Faulty Border*, which is denoted as E. Figure 1b illustrates a potential E.

Immediately from the above definition, for any given formula R, it follows that the risk values of all points on E are solely decided by their values of e_p. And immediately after Lemma 3.2, the faulty statement s_f is associated with the point (F, e_p^f) of E, where $0 \leq e_p^f \leq P$, as stated in the following lemma.

Lemma 3.4 (Location of faulty statement s_f)**.** The faulty statement s_f must be associated with a point (F, e_p^f) on E. And e_p^f can be any value between $[0, P]$.

Lemma 3.4 reflects a phenomenon in software testing called "Coincidental Correctness Test" (CCT). Ideally, the faulty statement s_f will produce $e_p = 0$, as executing s_f should result in a failure. CCTs are the tests that execute s_f but still pass. The number of CCT is equal to e_p^f, i.e. the value of e_p for s_f. There can be an arbitrary number of CCTs in a given test suite, and so is the value of e_p^f.

As a reminder, points (F, e_p^i) other than the one associated with s_f on E are associated with correct statements, where $n_f^i = F \wedge 0 \leq e_p^i \leq P \wedge e_p^i \neq e_p^f$. Depending on the adopted formula, the risk values of such points can be either greater than, equal to, or smaller than that of point (F, e_p^f), i.e. the point associated with s_f.

Lemma 3.5. For a given program and a test suite, the point of E, with which s_f is associated, may also be associated with other correct statements s_i having $(F, e_p^i) = (F, e_p^f)$. These statements share the same risk values as that of s_f, regardless of the selection of the formula.

Lemma 3.5 reflects another common phenomenon in software testing, that is, correct statements s_i may still have $e_f^i = F$, and their e_p^i could be either greater than, equal to or smaller than e_p^f of the faulty statement s_f and so are their risk values. An example of the faulty border can be found in Figure 2.

3.2 *Maximality in* $\mathbb{R}$

First, let us present the definition of the maximality with respect to $\mathbb{R}$.

Definition 3.6. Maximality. A risk evaluation formula R is said to be a maximal formula in $\mathbb{R}$ if, for any formula $R' \in \mathbb{R}$ such that $R' \neq R \wedge R' \rightarrow R$, it also holds that $R' \leftrightarrow R$. Let $\mathbb{M}$ be the set of formulas that are maximal with respect to $\mathbb{R}$.

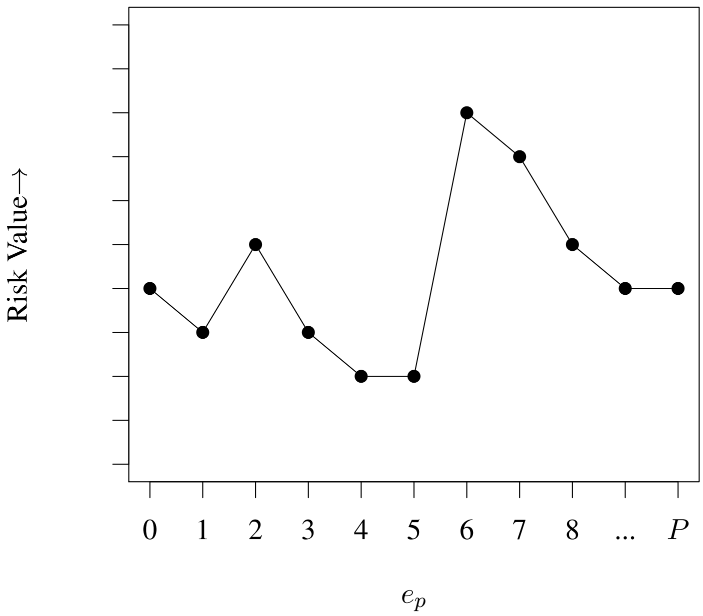


Figure 2: Example of faulty border of $\overline{R}$

Definition 3.7. Ranking. Given a formula R, use $o_p^{i,j} =< n_p^i, n_p^j, op >$ to denote the relation between the risk scores of two distinct points (F, n_p^i) and (F, n_p^j) on the faulty border. Given that $n_p^i < n_p^j$, op can be either ">" (i.e., $R(F, n_p^i) > R(F, n_p^j)$), "<" (i.e., $R(F, n_p^i) < R(F, n_p^j)$), or "=" (i.e., $R(F, n_p^i) = R(F, n_p^j)$).

Let P_R denote the set of $o_p^{i,j}$ based on Definition 3.7. P_R effectively captures the ranking between the statements that belong to E, by collecting the relations between risk scores of each pair of distinct points on the faulty border. Let U_R denote the set of points outside the faulty border which have risk scores higher than or equal to those of some points (F, e_p^i) on the faulty border, for formula R.

With all the above preliminary, let us now turn to the analysis of the maximality for all formulæ in $\mathbb{R}$.

Lemma 3.8. For any formula $R \in \mathbb{R}$, if $U_R \neq \emptyset$, then $R \notin \mathbb{M}$.

Proof. The proof shows that, if $U_R \neq \emptyset$, then there exists $R' \in \mathbb{R}$ such that $R' \rightarrow R$ but $R \nrightarrow R'$. First, let us construct $R' \in \mathbb{R}$ such that $R' \rightarrow R$. Assume that U_R is non-empty. Let R' be defined as follow:

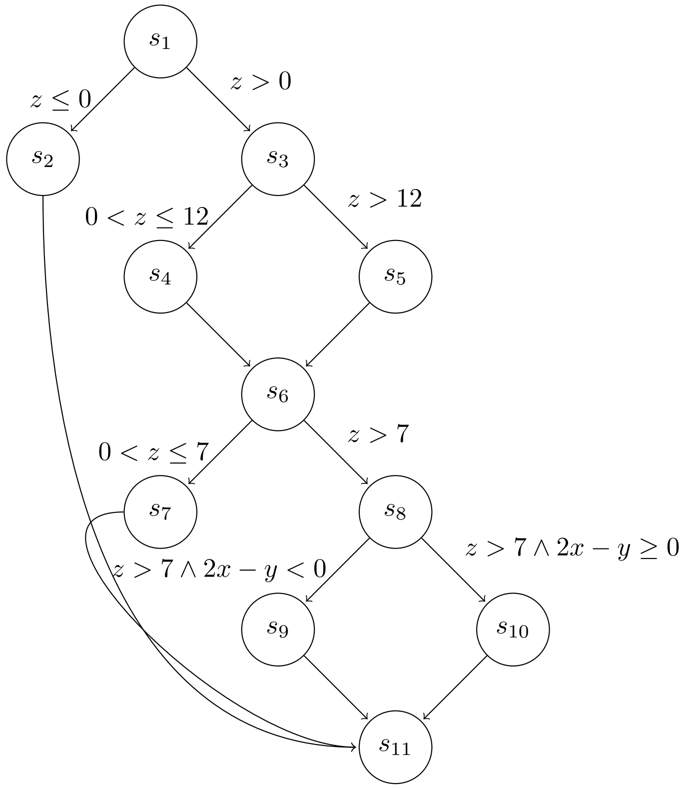
$$R' = \begin{cases} R & \text{if } e_f = F \\ R - (C_1 - C_2 + 1) & \text{otherwise} \end{cases}$$

where C_1 is the highest risk value of R for all points outside E, while C_2 is the lowest risk value of R for all points on E. By the definition of R', any point outside E has risk value lower than those of the points in E, which means all statements associated with points outside E have risk values lower than that of s_f.

Let $U_{R'}$ denote the sets of points outside the faulty border which have risk values higher than or equal to those of some points (F, e_p^i) on the faulty border, for formula R'. By definition, R' assigns identical risk values to points on the faulty border as R, while ensuring that $U_{R'} = \emptyset$.

```
        void foo(double x, double y, double z) {
s1 :      if(z <= 0){
s2 :        // s2
          } else {
s3 :        if (z <= 12) {
s4 :          // s4
            } else {
s5 :          // s5
            }
s6 :        if (z <= 3) {
s7 :          // s7
            } else {
s8 :          if (2 * x − y < 0) { //faulty, should be
                  : if (x − y < 0)
s9 :            // s9
              } else {
s10 :           // s10
              }
            }
          }
s11 :     return; // s11
        }
```

(a) Source Code



(b) Control Flow

Figure 3: Sample program: the faulty statement s_f is s_8.

- Consider the statements associated with E: these statements will be assigned to the same set division by both R and R', for any pair of program and test suite.
- Consider the statements associated with points outside E. For formula R', since these points (including those in U_R) always have risk values lower than that of s_f on E, the corresponding statements belong to $S_A^{R'}$. However, for formula R, since $U_R \neq \emptyset$, some statements corresponding to points outside E belong to either S_B^R, S_F^R, and S_A^R.

Summarizing the above two cases, we have $S_B^{R'} \subseteq S_B^R$ and $S_A^R \subseteq S_A^{R'}$. Following Theorem 2.6, $R' \rightarrow R$.

Let us now turn to show that $R \nrightarrow R'$, by illustrating that it is possible for R' to produce a smaller Expense value than R. Since $U_R \neq \emptyset$, there exists L, a set of points on E whose risk values evaluated by R are not higher than any point in U_R. To show that R' can produce a smaller Expense value than R, it is sufficient to show that $\bar{\sigma}(s_f) \in L$ while $U_R \neq \emptyset$. However, both L and U_R are specific to the choice of R. In order not to lose generality, therefore, let us show that it is possible to construct a program and a test suite such that $\bar{\sigma}(s_f)$ can be placed anywhere on E, and another statement $\bar{\sigma}(s_i)$ can be placed anywhere in $I_f \times I_p - E$, independently from each other[3]. Figure 3 illustrates such a program: the feasibility of the construction of the test suite is described in Example 1 in Appendix.

With such a program and a test suite, any statement associated with points outside U_R always have the same relative ranking to s_f in R and R'. For all statements associated with U_R, formula R' will rank them below s_f. However, with R:

- statements that are associated with U_R and have risk values higher than that of s_f, are always ranked before s_f by R.

[3]Given a specific R such that $U_R \neq \emptyset$, this allows us to place $\bar{\sigma}(s_f) \in L$ and $\bar{\sigma}(s_i) \in U_R$.

- statements that are associated with U_R and have risk values equal to that of s_f, will be tied together with s_f by R. However, it is possible to have a consistent tie-breaking scheme which ranks parts or even all of these statements before s_f.

It is always possible to have statements associated with U_R ranked before s_f. Consequently, the Expense of R' is smaller than that of R. Therefore, $R \rightarrow R'$ does hold.

In conclusion, if R assigns point (e_f^j, e_p^j) outside E with risk value higher than, or equal to, that of at least one point (F, n_p^i) on E, there always exists another formula R' for which $R' \rightarrow R$ holds but $R \rightarrow R'$ does not hold. Therefore, following Definition 3.6, R cannot be a maximal formula. □

Given two distinct risk evaluation formulæ, R_1 and R_2, let P_{R_1} and P_{R_2} denote the set of $o_p^{i,j}$ for all pairs of distinct points (F, e_p^i) and (F, e_p^j) on the faulty border (where $e_p^i < e_p^j$), for R_1 and R_2, respectively. Let U_{R_1} and U_{R_2} denote the sets of points outside the faulty border which have risk values higher than, or equal to, those of some points (F, e_p^i) on the faulty border, for formula R_1 and R_2, respectively.

Lemma 3.9. If $U_{R_1} = U_{R_2} = \emptyset$ and $P_{R_1} = P_{R_2}$, it follows that $R_1 \leftrightarrow R_2$.

Proof. Consider the following two cases.

- For statements associated with the faulty border E, since $P_{R_1} = P_{R_2}$, then for each pair of these statements, the relation between their risk values is always the same in R_1 and R_2. As a consequence, these statements have the same relative order with respect to s_f (which is associated with one point on the faulty border) between R_1 and R_2, and hence belong to the same set-division for R_1 and R_2 with any pair of program and test suite.

- For statements associated with points outside E, since both U_{R_1} and U_{R_2} are empty, these statements always have risk values lower than that of the faulty statement s_f (which is associated with one point on the faulty border), therefore these statements belong to both $S_A^{R_1}$ and $S_A^{R_2}$.

In summary, we have $S_B^{R_1} = S_B^{R_2}$, $S_F^{R_1} = S_F^{R_2}$ and $S_A^{R_1} = S_A^{R_2}$. Following Theorem 2.7, $R_1 \leftrightarrow R_2$. □

Lemma 3.10. If $U_{R_1} = U_{R_2} = \emptyset$ but $P_{R_1} \neq P_{R_2}$, we have $R_1 \nrightarrow R_2$ and $R_2 \nrightarrow R_1$.

Proof. Since $P_{R_1} \neq P_{R_2}$, there must exist at least one pair of points on the faulty border $((F, e_p^i), (F, e_p^j))$ (where $e_p^i < e_p^j$), such that $< e_p^i, e_p^j, op_1 > \in P_{R_1} \wedge < e_p^i, e_p^j, op_2 > \in P_{R_2} \wedge op_1 \neq op_2$. It is sufficient to consider the following two cases because other cases can be transformed to these two cases by swapping R_1 and R_2:

- Consider the case that $R_1(F, e_p^i) < R_1(F, e_p^j)$ and $R_2(F, e_p^i) > R_2(F, e_p^j)$.

  With the program shown in Figure 3, it is possible to construct a test suite, such that e_f^4, e_f^5, e_f^9 and e_f^{10} are smaller than F. (As a reminder, it always holds that $e_f^2 = e_f^7 = 0$). While for s_1, s_3, s_6, s_8 (s_f) and s_{11}, whose e_f values are all equal to F, we have $e_p^f = e_p^i < e_p^1 = e_p^3 = e_p^6 = e_p^{11} = e_p^j$[4]. Then, for R_1, we have s_1, s_3, s_6 and s_{11} ranked before s_f and other statements ranked after s_f. However, for R_2, we have s_f ranked at the top of the whole list. Therefore, the Expense of R_2 is lower than that of R_1.

  On the other hand, it is also possible to construct another test suite, such that e_f^4 and e_f^5 are both smaller than F, but e_f^9 is equal to F. (Correspondingly, $e_f^{10} = 0$). For s_1, s_3, s_6, s_8 (s_f), s_9 and s_{11}, whose e_f values are all equal to F, we have $e_p^9 = e_p^i < e_p^1 = e_p^3 = e_p^6 = e_p^f = e_p^{11} = e_p^j$[5]. Then, for

[4]For the feasibility of this test suite, please refer to **Test Suite A** in Example 5 of the Appendix.
[5]For the feasibility of this test suite, please refer to **Test Suite B** in Example 5 of the Appendix.

R_1, s_1, s_3 s_6, s_f and s_{11} are tied together at the top of the whole list, before s_9. However, for R_2, s_9 is ranked at the top, immediately followed by s_1, s_3 s_6, s_f and s_{11} that are tied together. Therefore, with a consistent tie-breaking scheme, the Expense of R_1 is lower than that of R_2.

In summary, for the case that $R_1(F, e_p^i) < R_1(F, e_p^j)$ while $R_2(F, e_p^i) > R_2(F, e_p^j)$, it is always possible to find examples to demonstrate $R_1 \nrightarrow R_2$ and $R_2 \nrightarrow R_1$

- Consider the case that $R_1(F, e_p^i) < R_1(F, e_p^j)$ and $R_2(F, e_p^i) = R_2(F, e_p^j)$.

  With the program shown in Figure 3, it is possible to construct a test suite, such that $e_f^4 = F$ (correspondingly, $e_f^5 = 0$), while e_f^9 and e_f^{10} are smaller than F. Then, for s_1, s_3, s_4, s_6, s_8 (s_f), and s_{11}, whose e_f values are all equal to F, it follows that $e_p^4 = e_p^i < e_p^1 = e_p^3 = e_p^6 = e_p^f = e_p^{11} = e_p^j$[6]. Then, for R_1, s_1, s_3, s_6, s_f, and s_{11} are tied together at the top of the ranking, before s_4. However, for R_2, s_1, s_3, s_4, s_6, s_f, and s_{11} are tied together at the top of the entire ranking. Since the number of tied statements are different, the Expense now depends on the tie-breaking scheme, without any guarantee of clear dominance of one formula. For example, if the original order of the statements is used as the tie-breaker, R_1 yields a lower Expense value than R_2; if the reverse of the original order is adopted, the opposite would follow.

  On the other hand, it is also possible to construct another test suite, such that e_f^4, e_f^5, e_f^9 and e_f^{10} are smaller than F. Then, for s_1, s_3, s_6, s_8 (s_f) and s_{11} whose e_f values are all equal to F, we have $e_p^f = e_p^i < e_p^1 = e_p^3 = e_p^6 = e_p^{11} = e_p^j$. (For the feasibility of this test suite, please refer to **Test Suite A** in Example 5 of the Appendix.) Then, for R_1, we have s_1, s_3, s_6 and s_{11} tied together at the top of the whole list before s_f, and other statements ranked after s_f. However, for R_2, we have s_1, s_3, s_6, s_f and s_{11} tied together at the top of the whole list. Since the number of tied statements are different, the Expense now depends on the tie-breaking scheme, without any guarantee of clear dominance of one formula. For example, if the original order of the statements is used as the tie-breaker, R_2 yields a lower Expense value than R_1; if the reverse of the original order is adopted, the opposite would follow.

  In summary, for the case that $R_1(F, e_p^i) < R_1(F, e_p^j)$ while $R_2(F, e_p^i) = R_2(F, e_p^j)$, it is possible to demonstrate that $R_1 \nrightarrow R_2$ and $R_2 \nrightarrow R_1$.

In conclusion, for any two formulæ whose U_{R_1} and U_{R_2} are both $\emptyset$, but $P_{R_1} \neq P_{R_2}$, it follows that $R_1 \nrightarrow R_2$ and $R_2 \nrightarrow R_1$. □

With above preliminaries, let us now turn to the complete analysis of the maximal and greatest formulæ of $\mathbb{R}$.

Proposition 3.11. A formula R is a maximal formula of in $\mathbb{R}$ **if and only if** U_R is empty.

Proof. First, it holds that if R is a maximal formula, then $U_R = \emptyset$. This follows immediately after Lemma 3.8.

Second, let us turn to proving that if $U_R = \emptyset$, R is a maximal formula. Assume that $U_R = \emptyset$. Then, for any distinct formula R', let $P_{R'}$ denote the set of $o_p^{i,j}$ for all pairs of distinct points (F, n_p^i) and (F, n_p^j) on the faulty border (where $n_p^i < n_p^j$) and $U_{R'}$ denote the sets of points outside the faulty border which have risk values higher than or equal to those of some points (F, n_p^i) on the faulty border, for formula R'. There are following cases.

- Consider the case that $U_{R'} \neq \emptyset$. As illustrated in the proof of Lemma 3.8, it is always possible to construct another formula R'', such that $U_{R''} = \emptyset$, $R'' \rightarrow R'$ and $R' \nrightarrow R''$. If $P_{R''} = P_R$, after Lemma 3.9, $R'' \leftrightarrow R$, and, consequently, $R' \nrightarrow R$. Otherwise, if $P_{R''} \neq P_R$, after Lemma 3.10, $R \nrightarrow R''$ and $R'' \nrightarrow R$. As a consequence, $R' \nrightarrow R$.

[6] For the feasibility of this test suite, please refer to **Test Suite C** in Example 5 of the Appendix.

- Consider the case that $U_{R'} = \emptyset$. Similar to the above analysis, if $P_{R'} = P_R$, after Lemma 3.9, $R' \leftrightarrow R$. Otherwise, if $P_{R'} \neq P_R$, after Lemma 3.10, $R \nrightarrow R'$ and $R' \nrightarrow R$.

In summary, if $U_R = \emptyset$, then for any formula R', we have either $R' \leftrightarrow R$ or $R' \nrightarrow R$. After Definition 3.6, R is a maximal formula. □

With Proposition 3.11, which is a necessary and sufficient condition for a maximal formula of $\mathbb{R}$, there is a simple method to convert any given non-maximal formula into a maximal formula, which is effectively described in the proof of Lemma 3.8. Given any formula R which has a non-empty U_R (i.e. R is non-maximal according to Proposition 3.11), we can always convert R into a maximal formula R', where R' assigns identical risk values to points on the faulty border as R, but assigns, to all points in U_R, a constant C whose value is smaller than the risk value of any point on the faulty border.

Now, let us re-visit the five maximal formulæ of the 50 investigated formulæ. After Proposition 3.11, it is possible to show that two of them (ER'_1 and ER_5) are maximal but not greatest formulæ, while three of them (GP2, GP3 and GP19) are not maximal with respect to $\mathbb{R}$, as follows:

Corollary 3.12. ER'_1 and ER_5 are maximal formulæ of $\mathbb{R}$.

Proof. For any formula R in ER'_1 or ER_5, $U_R = \emptyset$. After Proposition 3.11, formulæ in ER'_1 and ER_5 are maximal formulæ of $\mathbb{R}$. □

Corollary 3.13. GP2, GP3 and GP19 are not maximal to all formulæ in $\mathbb{R}$.

Proof. Consider the program in Figure 3. The following three test suites can be constructed:

1. For GP2:

Construct a test suite that satisfies the following: $F = 2$, $P = 9$, s_8 (s_f) satisfies $e_f^f = 2 = F \wedge e_p^f = 5 < P$, and s_9 satisfies $e_f^9 = 1 < F \wedge e_p^9 = 4 < e_p^f$. Then, following the definition of GP2 (which is $2(e_f + \sqrt{n_p}) + \sqrt{e_p}$), the following risk evaluation values are obtained: $GP2(s_8) = 13$, which is smaller than $GP2(s_9) = 14$. Since s_f is on E, this shows that there can exist points outside E with risk values higher than that of the point on E. Following Proposition 3.11, GP2 is not the maximal with respect to $\mathbb{R}$[7].

2. For GP3:

Construct a test suite that satisfies the following: $F = 2$, $P = 30$, s_f satisfies $e_f^f = 2 = F \wedge e_p^f = 25 < P$, and s_9 satisfies $e_f^9 = 1 < F \wedge e_p^9 = 25 = e_p^f$. Then, following the definition of GP3 (which is $\sqrt{|e_f^2 - \sqrt{e_p}|}$), the following risk evaluation values are obtained: $GP3(s_f) = 1$, which is smaller than $GP3(s_9) = 2$. Since s_f is on E, this shows that there can exist points outside E with risk values higher than that of the point on E. Following Proposition 3.11, GP3 is not the maximal to all formulæ in $\mathbb{R}$[8].

3. For GP19:

Construct a test suite that satisfies the following: $F = 5$, $P = 50$, s_f satisfies $e_f^f = 5 = F \wedge e_p^f = 1 < P$, and s_4 satisfies $e_f^4 = 4 < F \wedge e_p^f < e_p^4 = 49 < P$. Then, following the definition of GP19 (which is $e_f\sqrt{|e_p - e_f + n_f - n_p|}$), the following risk evaluation values are obtained: $GP19(s_f) = 5\sqrt{5}$, which is smaller than $GP19(s_4) = 12\sqrt{5}$. Since s_f is on E, this shows that there can exist points outside E with risk values higher than that of the point on E. Following Proposition 3.11, GP19 is not the maximal to all formulæ in $\mathbb{R}$[9].

[7] The feasibility of this scenario is analysed in Example 2 of the Appendix.
[8] The feasibility of this scenario is analyzed in Example 3 of the Appendix.
[9] The feasibility of this scenario is analyzed in Example 4 of the Appendix.

□

With Proposition 3.11, it becomes possible to identify maximal formula with respect to $\mathbb{R}$. Furthermore, within these maximal formulæ, we are interested in whether there exists the greatest formulæ and have the following conclusion.

3.3 Greatest Formulæ in $\mathbb{R}$: The Non-Existence Proof

A greatest formula in $\mathbb{R}$ is the formula that is better than any other formulæ in $\mathbb{R}$. It is formally defined as follows:

Definition 3.14. Greatest Formula. A risk evaluation formula R is said to be a *greatest* formula in $\mathbb{R}$ if, for any formula $R' \in \mathbb{R} \wedge R' \neq R$, it holds that $R \rightarrow R'$.

Let us now turn to the greatest formula, or, in fact, proving the lack of thereof.

Proposition 3.15. There is no formula which is greatest against the set of all formulæ, $\mathbb{R}$.

Proof. Assume that there exists a greatest formula R_g. Let P_{R_g} denote the set of $o_p^{i,j}$ for all pairs of distinct points (F, e_p^i) and (F, e_p^j) on the faulty border (where $e_p^i < e_p^j$) and U_{R_g} denote the sets of points outside the faulty border which have risk values higher than or equal to those of some points (F, e_p^i) on the faulty border, for formula R_g. After Proposition 3.11, $U_{R_g} = \emptyset$.

Consider the two maximal groups of formulæ ER_1' and ER_5, which have been proved to be non-equivalent to each other [18]. Let $U_{ER_1'}$ and U_{ER_5} denote the sets of points outside the faulty border which have risk values higher than or equal to those of some points (F, e_p^i) on the faulty border, for formulæ in ER_1' and ER_5, respectively. Let $P_{ER_1'}$ and P_{ER_5} denote the set of $o_p^{i,j}$ for all pairs of distinct points (F, e_p^i) and (F, e_p^j) on the faulty border (where $e_p^i < e_p^j$), for formulæ in ER_1' and ER_5, respectively. According to Corollary 3.12, it follows that $U_{ER_1'} = U_{ER_5} = \emptyset$ and $P_{ER_1'} \neq P_{ER_5}$. Thus, there are three possible cases for P_{R_g}, as follows:

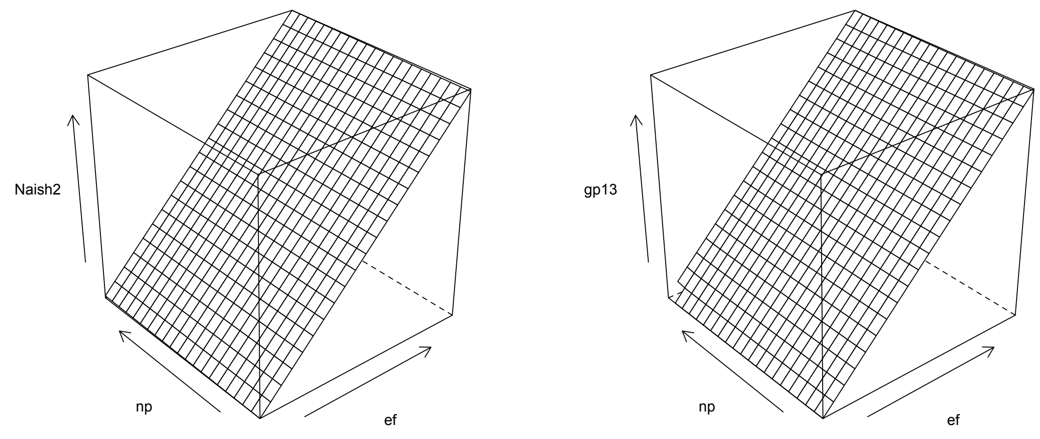
- Case 1: $P_{R_g} = P_{ER_1'}$. Then it follows that, for ER_5, $U_{ER_5} = U_{R_g} = \emptyset \wedge P_{ER_5} \neq P_{R_g}$.
- Case 2: $P_{R_g} = P_{ER_5}$. Then it follows that, for ER_1', $U_{ER_1'} = U_{R_g} = \emptyset \wedge P_{ER_1'} \neq P_{R_g}$.
- Case 3: $P_{R_g} \neq P_{ER_1'}$ and $P_{R_g} \neq P_{ER_5}$. Then it follows that, both for ER_1' and ER_5, $U_{ER_1'} = U_{R_g} = \emptyset \wedge P_{ER_1'} \neq P_{R_g} \wedge U_{ER_5} = U_{R_g} = \emptyset$ but $P_{ER_5} \neq P_{R_g}$.

For any of the above cases, it is possible to construct another formula R' such that $U_{R'} = U_{R_g} = \emptyset$ and $P_{R'} \neq P_{R_g}$. After Lemma 3.10, we have $R' \nrightarrow R_g$ and $R_g \nrightarrow R'$. After Definition 3.14, R_g cannot be the greatest formula. □

4 Visualising the Insights

The spectral coordinate $\bar{\sigma}$, introduced in Section 3.1, provides an intuitive way to visualise SBFL formulæ. Since neither the size of the test suite nor the exact number of passing and failing test cases is known, we normalise the visualisation with $P = 100$ and $F = 100$. In addition, without affecting generality, we flip the P axis so that the visualisation depicts n_p instead of e_p: this is because, intuitively, n_p correlates better with the risk value of a statement (the higher n_p is, the more suspicious the corresponding statement is, because it means that test cases are more likely to pass when not executing the statement). The grids on the plots are separated by the margin of 5.

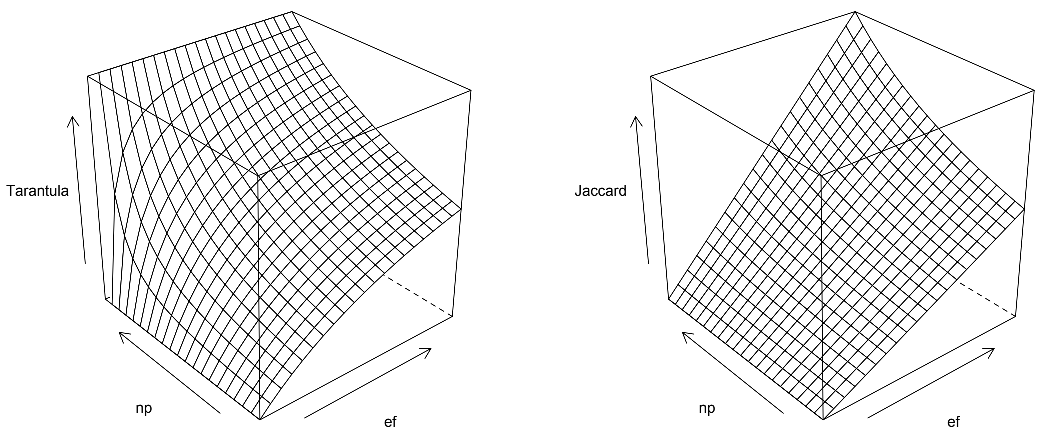
Figure 4 contains visualisations of Naish2 and GP13, two equivalent formulas. The visualisation provides an intuitive understanding of the equivalence. Both formulas assign any points outside the faulty edge E



(a) Visualisation of Naish2 (b) Visualisation of GP13

Figure 4: Visualisation of two equivalent SBFL formulæ, Naish2 and GP13

with lower risk value than the points on E. Along E, the risk value slowly increases monotonically as n_p increases: therefore, given the same spectrum data, both formulæ will rank the statements that belong to E in the identical order.



(a) Visualisation of Tarantula (b) Visualisation of Jaccard

Figure 5: Visualisation of SBFL formulæ: Tarantula and Jaccard

Figure 5 contains visualisations of Tarantula and Jaccard. The Tarantula plot provides an intuitive understanding on why it is not a maximal formula: to many points with $n_p \approx P$ and $e_f \approx 0$, Tarantula assigns significantly high risk values: unless the faulty statement satisfies $n_p = 0$, it may be ranked below some of these points.

The visualisation of Jaccard provides another intuitive understanding into a recent observation in SBFL literature. Qi et al. [15] reported that, when used in conjunction with automated bug patching technique GenProg [16], Jaccard outperformed Naish2 by leading GenProg to more generated patches, despite being dominated by Naish2. The fact that Jaccard is dominated by Naish2 can be seen by the fact that some points outside E can be assigned with higher risk value than other points on E. However, when the faulty statement has sufficiently high n_p, the difference in risk values between the faulty statement and others can be much larger with Jaccard than in Naish2 (which only allows very small change of risk value along E). Moon et al. introduced a new evaluation metric for SBFL, called Locality Information Loss (LIL), which explained that the bigger difference is more helpful when risk values are used as mutation rates [10]. The visualisation supports this explanation.

5 Conclusions and Future Work

Spectrum Based Fault Localisation (SBFL) has received significant amount of attention over the past decade. The focus of the research mainly has been the design of new risk evaluation formulæ that would outperform the existing ones. The evaluation has been of empirical nature, until theoretical analysis began recently. This paper presents the proof that there does not exist the greatest formula, i.e. the one that is better than all other formulæ.

The proof has a significant implication on the research of SBFL. Pursuing the greatest formula is no longer a viable research goal. Concerning SBFL, the future work will be encouraged to consider specialisation, i.e. designing formulæ that are effective in certain contexts, such as a specific project or a particular type of faults. In the wider context, the proof illustrates the limitations of the spectrum based approaches, and encourages fault localisation techniques to consider signals other than program spectrum.

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Appendix

Analysis of the Example Program

In the proof, the program in Figure 3 has been used as an example. The program accepts three independent real numbers: x, y and z. The statement s_8 contains a fault: the correct predicate should be "**if**(x-y <0)", not "**if**(2x-y<0)" and, therefore, is denoted by s_f. Figure 6 shows the corresponding regions of failure inducing inputs in the space of all possible inputs.

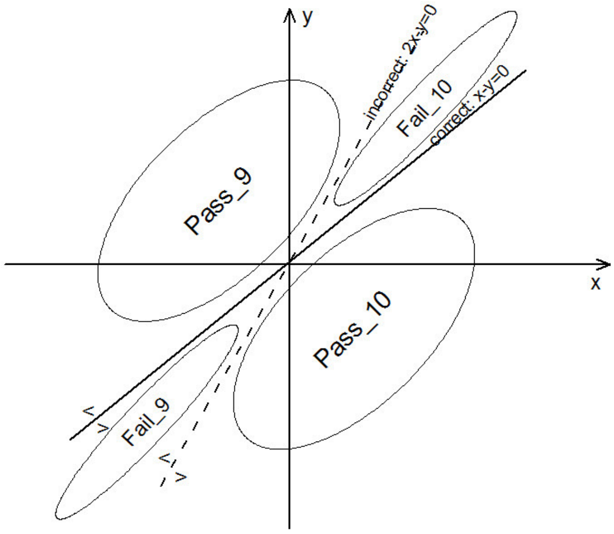

Figure 6: Sample program

First, consider statements s_9 and s_{10}:

- Any test case $t_i = (x_i, y_i, z_i)$ such that $z_i > 7 \wedge (x_i, y_i) \in Fail_9$ will cover s_9 and fail. Let the number of such test cases be e_f^9.
- Any test case $t_i = (x_i, y_i, z_i)$ such that $z_i > 7 \wedge (x_i, y_i) \in Pass_9$ will cover s_9 and pass. Let the number of such test cases be e_p^9.
- Any test case $t_i = (x_i, y_i, z_i)$ such that $z_i > 7 \wedge (x_i, y_i) \in Fail_{10}$ will cover s_{10} and fail. Let the number of such test cases be e_f^{10}.
- Any test case $t_i = (x_i, y_i, z_i)$ such that $z_i > 7 \wedge (x_i, y_i) \in Pass_{10}$ will cover s_{10} and pass.Let the number of such test cases be e_p^{10}.

Since s_9 and s_{10} are the **true** and **false** branches of the **if** statement in s_f, only one of these two statements are executed by any test case. Consequently, $e_f^9 = n_f^{10}$, $n_f^9 = e_f^{10}$, $e_p^9 = n_p^{10}$, and $n_p^9 = e_p^{10}$. There is *no* constraint in selecting test cases from each of the above four regions, and generating test cases from these regions is independent from each other. Therefore, by adjusting the number of test cases in each region, it is possible to have values of e_f^9, e_p^9, e_f^{10} and e_p^{10}.

Next, consider statement s_8 (i.e. s_f). It is possible to have any number of passing (i.e. e_p^f) and failing (i.e. $e_f^f = F$) test cases by adjusting the above four sets of test cases, because we have $e_f^9 + e_f^{10} = e_f^f = F$ and $e_p^9 + e_p^{10} = e_p^f$.

Then, let us consider statement s_7. Any test case $t_i =< x_i, y_i, z_i >$ such that $0 < z_i \leq 7$ will cover s_7 and pass: e_p^7 is equal to the number of such test cases, while $e_f^7 = 0$. It is possible to have any number of such test cases. Therefore, by adjusting this number, it is possible to assign any value to e_p^7.

Now, consider statements s_6, s_{11} and s_3, whose e_f values are identical to each other, and so are their e_p. It can be seen from Figure 3 that $e_f^6 = e_f^{11} = e_f^3 = e_f^f = F$ and $e_p^6 = e_p^3 = e_p^{11} = (e_p^f + e_p^7)$. As a reminder, according to the above analysis, e_p^f and e_p^7 can be assigned with any values independently.

Next, consider statements s_4 and s_5. As shown in Figure 3, any test case $t_i =< x_i, y_i, z_i >$ such that $0 < z_i \leq 12$ will cover s_4. These test cases can be further categorised as following:

- Any test case $t_i =< x_i, y_i, z_i >$ such that $0 < z_i \leq 7$ will definitely continue to cover s_7 and thus always pass. The number of these test cases is equal to e_p^7.
- Any test case $t_i =< x_i, y_i, z_i >$ such that $7 < z_i \leq 12$ while $< x_i, y_i > \in Pass_9 \cup Pass_{10}$ will also pass. Let us denote the number of these test cases as $\overline{e_p^4}$.
- Any test case $t_i =< x_i, y_i, z_i >$ such that $7 < z_i \leq 12$ while $< x_i, y_i > \in Fail_9 \cup Fail_{10}$ will always fail. The number of these test cases is e_f^4.

It is not difficult to find that e_f^4 is the size of the subset of failing test cases that cover s_f and satisfy $7 < z_i \leq 12$. Consequently, it follows that $e_f^4 \leq F$. On the other hand, $e_p^4 = e_p^7 + \overline{e_p^4}$, where $\overline{e_p^4}$ is the size of the subset of passing test cases that cover and satisfy $7 < z_i \leq 12$. Thus, it also follows that $\overline{e_p^4} \leq e_p^f$. As a reminder, the values of e_p^7 and $\overline{e_p^4}$ can be decided independently. Therefore, e_p^4 can be either smaller than, equal to or greater than e_p^f.

While for s_5, any test case $t_i =< x_i, y_i, z_i >$ such that $z_i > 12$ will cover s_5. These test cases can be further categorised as following:

- Any test case $t_i =< x_i, y_i, z_i >$ such that $z_i > 12$ while $< x_i, y_i > \in Pass_9 \cup Pass10$ will always pass. The number of these test cases is e_p^5.
- Any test case $t_i =< x_i, y_i, z_i >$ such that $z_i > 12$ while $< x_i, y_i > \in Fail_9 \cup Fail10$ will always fail. The number of these test cases is e_f^5.

Note that e_p^5 is the size of the subset of passing test cases that cover s_f and satisfy $z_i > 12$, while e_f^5 is the size of the subset of failing test cases that cover s_f and satisfy $z_i > 12$. Thus, it follows that $e_f^5 \leq e_f^f = F$ and $e_p^5 \leq e_p^f$.

It should be noted that e_f^4 and e_f^5 are not independent. There is a constraint that $e_f^4 + e_f^5 = e_f^f = F$. However, there is no similar constraint on e_p^4 and e_p^5.

Next, let us consider s_2. As shown in Figure 3, any test case $t_i =< x_i, y_i, z_i >$ such that $z_i \leq 0$ will cover s_2 and pass: e_p^2 is equal to the number of such test cases, while e_f^2 is always 0. It is possible to have any number of such test cases. By adjusting this number, it is possible to assign any value to e_p^2.

Finally, let us consider s_1. The structure of the program dictates that $e_f^1 = e_f^f = F$ and $e_p^1 = (e_p^3 + e_p^2) = (e_p^f + e_p^7 + e_p^2) = P$. As analyzed above, it is possible to assign, independently, any values to e_p^f, e_p^7 and e_p^2. Consequently, e_p^1 (i.e. the number of total passed test cases P) can be any value that is no less than e_p^f.

Feasibility of Test Suites Used in Proofs

This section presents the analysis of the feasibility of the example test suites used in the proofs.

Example 1. With the program in Figure 3, the proof in Proposition 3.11 requires the construction of a test suite such that e_p^f and e_p^4 are any values, and $e_f^4 < F$. According the above discussion, for s_f, we can assign any value to e_p^f; while for s_4, e_f^4 can be any value within [0, F] and e_p^4 can be either smaller than, equal to or greater than e_p^f. Therefore, it is always possible to construct such a test suite.

Example 2. With the program in Figure 3, the proof for GP2 in Corollary 3.13 requires the construction of a test suite such that $F = 2$, $P = 9$, s_8 (s_f) has $e_f^f = 2 = F$ and $e_p^f = 5 < P$, and s_9 has $e_f^9 = 1 < F$ and $e_p^9 = 4 < e_p^f$. According to the above analysis, we can assign any value to e_p^f and any value to P that is no less than e_p^f. And for s_9, we can have any value of e_f^9 within [0, F] and any value of e_p^9 within [0, e_p^f]. Therefore, it is always possible to construct such a test suite.

Example 3. With the program in Figure 3, the proof for GP3 in Corollary 3.13 requires the construction of a test suite such that $F = 2$, $P = 30$, s_f has $e_f^f = 2 = F$ and $e_p^f = 25 < P$, and s_9 has $e_f^9 = 1 < F$ and $e_p^9 = 25 = e_p^f$. Similar to the analysis in Example 2, it is always possible to construct such a test suite.

Example 4. With the program in Figure 3, the proof for GP19 in Corollary 3.13 requires the construction of a test suite such that $F = 5$, $P = 50$, s_f has $e_f^f = 5 = F$ and $e_p^f = 1 < P$, and s_4 has $e_f^4 = 4 < F$ and $e_p^f < e_p^4 = 49 < P$. According to the above analysis, for s_f, we can assign any values to e_f^f (i.e. F) and e_p^f; while for s_4, e_f^4 can be any value within [0, F] and e_p^4 can be either smaller than, equal to or greater than e_p^f; and P can be any value that is no less than either e_p^4 or e_p^f. Therefore, it is always possible to construct such a test suite.

Example 5. Let us denote the e_p values of any two points on the faulty border E as e_p^L and e_p^H, where $e_p^L < e_p^H$. For the given program in Figure 3, the proof in Lemma 3.10 requires construction of two test suites, which are referred to as **Test Suite A**, **Test Suite B** and **Test Suite C** in the following discussion.

Test Suite A: we have e_f^4, e_f^5, e_f^9 and e_f^{10} smaller than F, $e_f^1 = e_f^3 = e_f^6 = e_f^f = e_f^{11} = F$, $e_p^f = e_p^L$ and $e_p^1 = e_p^3 = e_p^6 = e_p^{11} = e_p^H$. According to the above analysis, e_f^4, e_f^5, e_f^9 and e_f^{10} can all be less than F simultaneously. And the e_f values for s_1, s_3, s_6, s_f and s_{11} are always equal to F. Besides, for s_f, it is always possible to have any value of e_p^f; while it is always possible to have any equal value of e_p that is larger than e_p^f for s_1, s_3, s_6 and s_{11}. Therefore, this test suite is feasible.

Test Suite B: we have both e_f^4 and e_f^5 smaller than F, $e_f^1 = e_f^3 = e_f^6 = e_f^f = e_f^9 = e_f^{11} = F$, $e_p^9 = e_p^L$ and $e_p^1 = e_p^3 = e_p^6 = e_p^f = e_p^{11} = e_p^H$. As discussed above, it is always possible to have both e_f^4 and e_f^5 smaller than F. And it is also possible to assign the same value of e_f (i.e. F) to s_1, s_3, s_6, s_f, s_9 and s_{11}. Moreover, s_1, s_3, s_6, s_f and s_{11} are possible to have the same e_p value that is higher than s_9. As a consequence, this test suite is always feasible.

Test Suite C: we have e_f^9 and e_f^{10} smaller than F, $e_f^1 = e_f^3 = e_f^4 = e_f^6 = e_f^f = e_f^{11} = F$, $e_p^4 = e_p^L$ and $e_p^1 = e_p^3 = e_p^6 = e_p^f = e_p^{11} = e_p^H$. As discussed above, it is always possible to have both e_f^9 and e_f^{10} smaller than F. And it is also possible to assign the same value of e_f (i.e. F) to s_1, s_3, s_4, s_6, s_f and s_{11}. Moreover, s_1, s_3, s_6, s_f and s_{11} are possible to have the same e_p value that is higher than s_4. As a consequence, this test suite is always feasible.