

MiniSAT and SAT Encoding

CS402, Spring 2017
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Boolean Satisfiability

- The problem of determining if there exists an interpretation that satisfies a given propositional formula.
- One of the first problems proven to be NP-complete.
- Often referred to simply as SAT

Computational Complexity

- 2-SAT refers to SAT problems, whose clauses only contain at most 2 literals. 2-SAT can be solved in polynomial time.
- 3-SAT refers to SAT problems whose clauses contain at most three literals.

$l_1 \vee \dots \vee l_n$ can be rewritten into 3-SAT as

$$(l_1 \vee l_2 \vee x_2) \wedge (\neg x_2 \vee l_3 \vee x_3) \wedge (\neg x_3 \vee l_4 \vee x_4) \wedge \dots$$
$$\wedge (\neg x_{n-3} \vee l_{n-2} \vee x_{n-2}) \wedge (\neg x_{n-2} \vee l_{n-1} \vee l_n)$$

Equisatisfiable

- Two formulas are equisatisfiable if the first formula is satisfiable whenever the second is, and vice versa.
- This is different from logical equivalence, which states that two formulas have the same models; equisatisfiable formulas have different models.
- 3-SAT conversion is equisatisfiable.

MiniSAT

- Award winning open-source SAT solver
- <http://minisat.se>



The screenshot shows the homepage of the MiniSAT project. The main heading is "The MiniSat Page" in a stylized blue font, with "by Niklas Een, Niklas Sörensson" written below it. A navigation menu on the left includes links for Main, MiniSat, MiniSat+, SatELite, Papers, Authors, and Links. The main content area is titled "INTRODUCTION" and describes MiniSAT as a minimalistic, open-source SAT solver. It mentions that MiniSAT is used in various projects and was recently awarded in the SAT 2005 competition. A photograph of four trophies is shown. Below the introduction, there is a section for "Some key features of MiniSAT:" with three bullet points: "Easy to modify", "Highly efficient", and "Designed for integration". A "NEWS" section follows, listing recent updates and papers. The page is signed off by "Niklas & Niklas".

The MiniSat Page

by Niklas Een, Niklas Sörensson

INTRODUCTION

MiniSAT is a minimalistic, open-source SAT solver, developed to help researchers and developers alike to get started on SAT. It is released under the MIT licence, and is currently used in a number of projects (see "Links"). On this page you will find binaries, sources, documentation and projects related to MiniSAT, including the Pseudo-boolean solver MiniSAT+ and the CNF minimizer/preprocessor SatELite. Together with SatELite, MiniSAT was recently awarded in the three industrial categories and one of the "crafted" categories of the SAT 2005 competition (see picture).



Some key features of MiniSAT:

- **Easy to modify.** MiniSAT is small and well-documented, and possibly also well-designed, making it an ideal starting point for adapting SAT based techniques to domain specific problems.
- **Highly efficient.** Winning all the industrial categories of the SAT 2005 competition, MiniSAT is a good starting point both for future research in SAT, and for applications using SAT.
- **Designed for integration.** MiniSAT supports incremental SAT and has mechanisms for adding non-clausal constraints. By virtue of being easy to modify, it is a good choice for integrating as a backend to another tool, such as a model checker or a more generic constraint solver.

We would like to start a community to help distribute knowledge about how to use and modify MiniSAT. Any questions, comments, bugreports, bugfixes etc. should be directed to minisat@googlegroups.com. The archives can be browsed [here](#). The source code repository for MiniSAT 2.2 can be found at [github](#).

— Niklas & Niklas

NEWS

From newest to oldest...

- A [paper](#) on how to compute localization abstractions using the incremental interface of MiniSat. View this as an example of a non-trivial incremental SAT application. A key feature of the algorithm is the use of `analyzeFinal()` to get the subset of assumptions used in the UNSAT proof.
- Finally! A new release of MiniSat, [downloads/minisat-2.2.0.tar.gz](#). For more info, see the [release notes](#).
- Paper on [Cut-sweeping](#) added, a light-weight alternative to SAT-sweeping useful for preprocessing AIGs before applying SAT.
- Added slides for invited talk given by Niklas Een at FMCAD ([Linux/Cygwin/Windows](#))
- Paper on [efficient CNF generation](#) added ([slides](#) also available).
- Bug-fix in MiniSat+ for trivially unsatisfiable instances.
- MiniSat v.2.0 has been released.
- Bug-fix to the proof-logging version of MiniSat (thanks to Georg Weissenbacher for reporting the problem!).
- Bug-fix to the C-version of MiniSat.
- Paper on MiniSat+ added.
- Removed version 1.13 of MiniSat. It was an intermediate version that apparently was buggy.
- Patch for Visual Studio users added.
- Buggy v1.13 Cygwin binary replaced with working v1.14 binary. The bug did not affect the Linux version.
- Fixed the output of MiniSat+ so that it is flushed properly when `stdout` is redirected to a file.

Building MiniSAT

- Ubuntu (and probably other distros): download `minisat-2.2.0.tar.gz`, unzip, follow instructions.
- OS X: some portability patches required, unfortunately not very centrally documented. Instead, you can do `brew install minisat`.
- Windows: avoid for the purpose of coursework, as it requires cygwin. There are pre-built binaries but they are old (ver 1.14).

Minisat

- Every line starting with `c` contains comment.
- You should define the problem size with:
 - `p cnf [number of literals] [number of clauses]`
- Subsequent lines contain each individual conjunct; literals are numbers, negations are `-` signs. Each line should end with `0`.

$$(x_1 \vee \neg x_5 \vee x_4) \wedge (\neg x_1 \vee x_5 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee x_4)$$

c blah blah...

p cnf 5 3

1 -5 4 0

-1 5 3 4 0

-3 4 0

"test.in"

c blah blah...

p cnf 5 3

1 -5 4 0

-1 5 3 4 0

-3 4 0

\$ minisat test.in

=====[Problem Statistics]=====

Number of variables:	5
Number of clauses:	3
Parse time:	0.00 s
Eliminated clauses:	0.00 Mb
Simplification time:	0.00 s

=====[Search Statistics]=====

Conflicts	ORIGINAL			LEARNT			Progress
	Vars	Clauses	Literals	Limit	Clauses	Lit/Cl	

=====
=====

restarts : 1
conflicts : 0 (0 /sec)
decisions : 1 (0.00 % random) (1133 /sec)
propagations : 0 (0 /sec)
conflict literals : 0 (nan % deleted)
Memory used : 0.17 MB
CPU time : 0.000883 s

SATISFIABLE

\$ _

$$(x_1 \vee \neg x_5 \vee x_4) \wedge (\neg x_1 \vee x_5 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee x_4)$$

```
"test.in"  
c blah blah...  
p cnf 5 3  
1 -5 4 0  
-1 5 3 4 0  
-3 4 0
```

```
$ minisat test.in test.out  
...  
$ cat test.out  
SAT  
-1 -2 -3 4 -5 0  
$ _
```

```
"test.in"  
c blah blah...  
p cnf 5 3  
1 -5 4 0  
-1 5 3 4 0  
-3 4 0  
1 2 3 -4 5 0
```

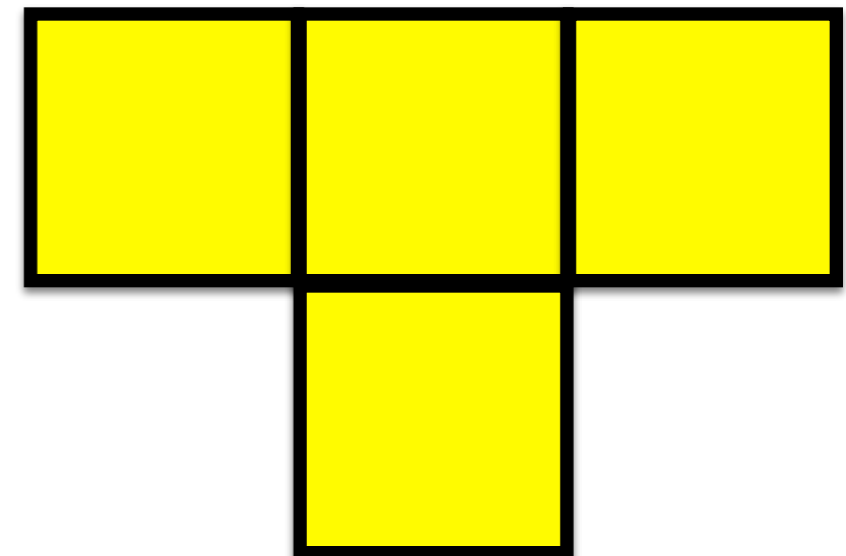
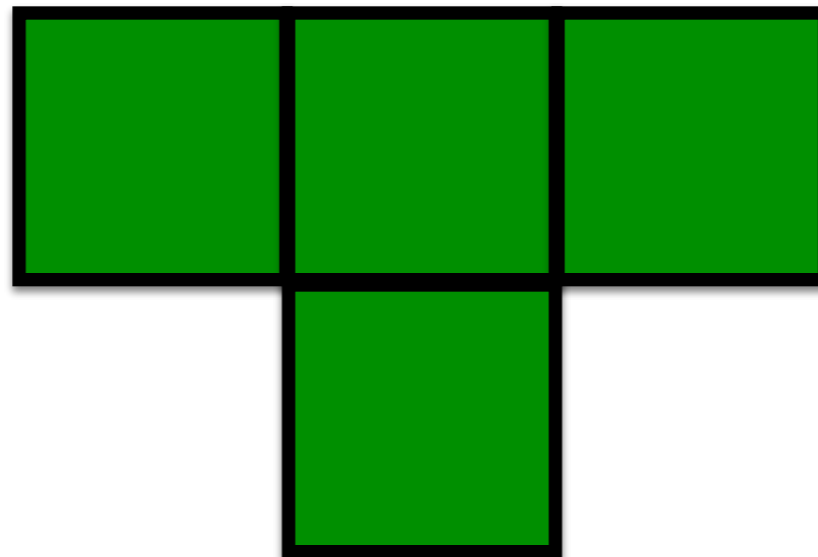
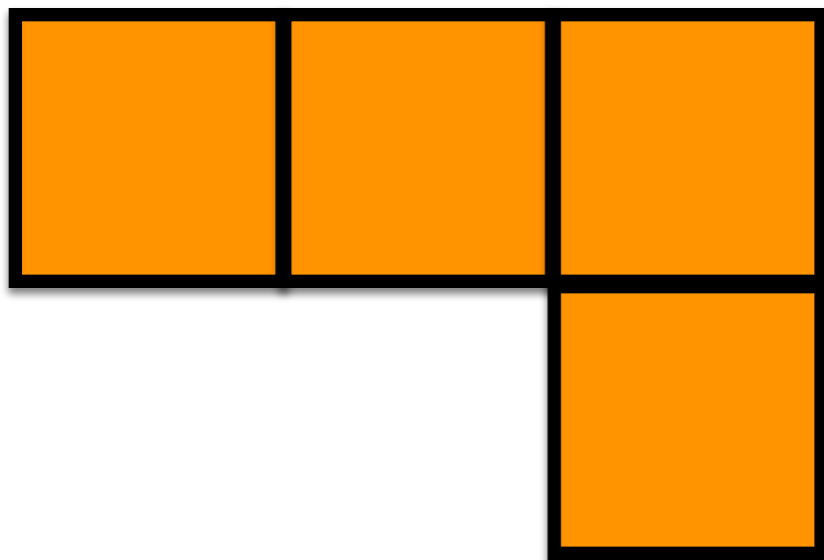
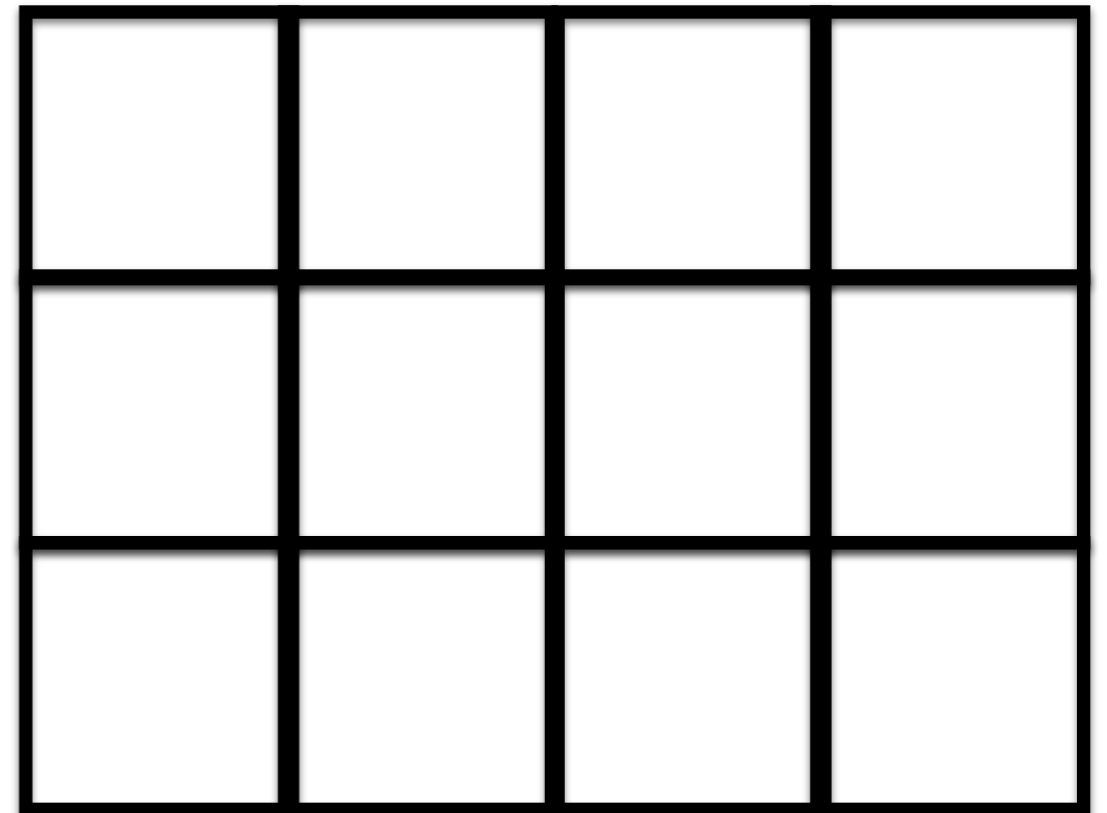
```
$ minisat test.in test.out  
...  
$ cat test.out  
SAT  
-1 2 -3 4 -5 0  
$ _
```

SAT Encoding

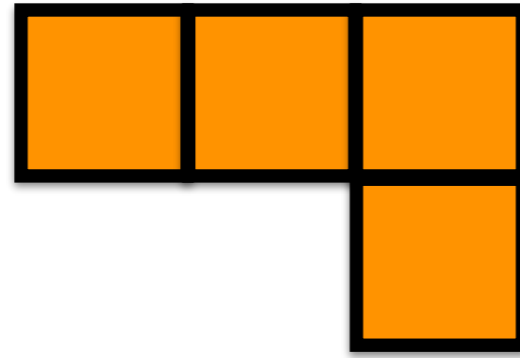
- Formulate a problem into SAT
- Solve the SAT
- Interpret the result back to the original domain

A Tetris-like Puzzle

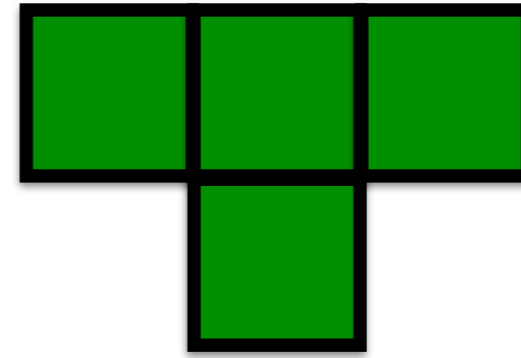
- Fill in the 4 by 3 grid using the given pieces.
- SAT...?



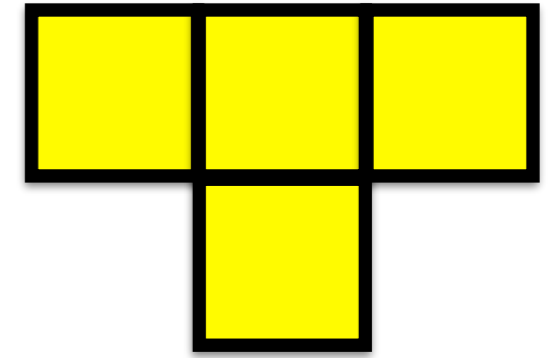
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)



1



2

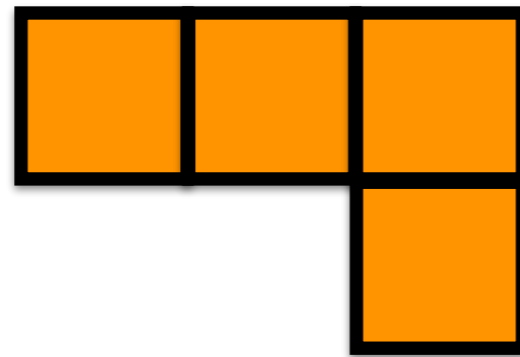


3

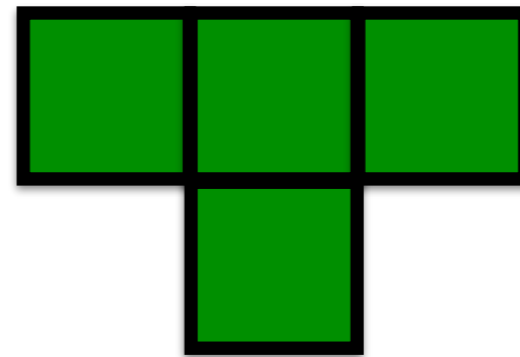
A correct solution should satisfy the following:

- Every grid has at least one piece on it.
- Every grid has at most one piece on it.
- Every piece is used at least once.
- Pieces cannot be broken.

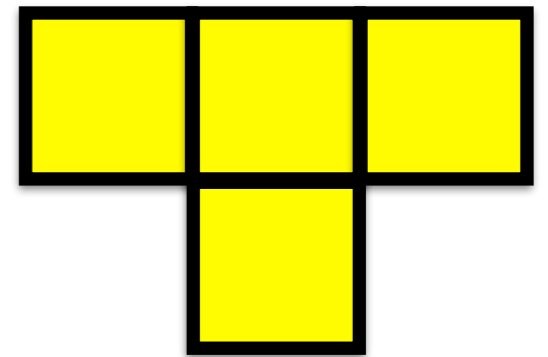
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)



1



2



3

Let x_{ijk} be the proposition that grid (i, j) .

First condition: “every grid has at least one piece on it”.

That is:

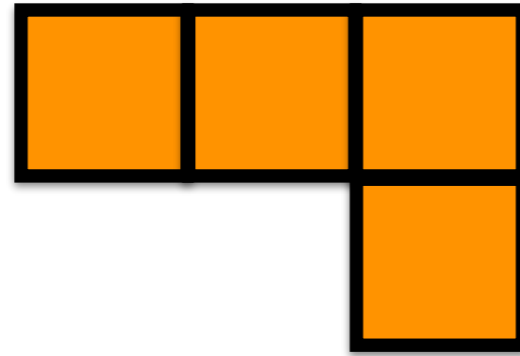
1. for grid (1, 1): $x_{111} \vee x_{112} \vee x_{113}$

2. for grid (1, 2): $x_{121} \vee x_{122} \vee x_{123}$

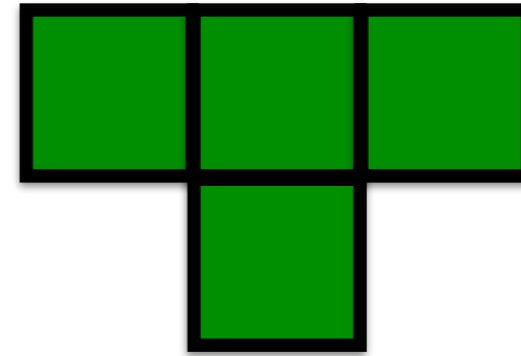
3. ...

$$\therefore (x_{111} \vee x_{112} \vee x_{113}) \wedge (x_{121} \vee x_{122} \vee x_{123}) \wedge \dots$$

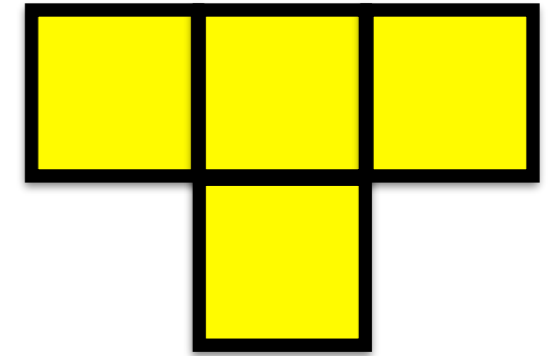
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)



1



2



3

Second condition: “every grid has at most one piece on it.”

That is, a single grid cannot be shared by two pieces. For example, grid (1, 1) cannot be shared by any pair of pieces.

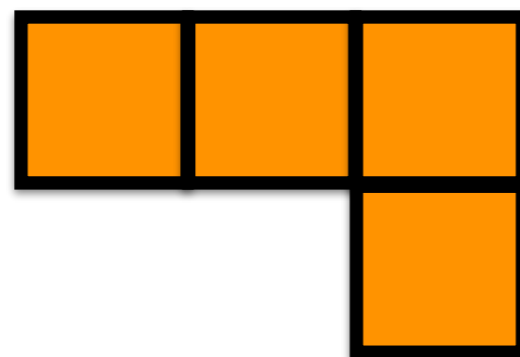
$$1. \neg(x_{111} \wedge x_{112}) = \neg x_{111} \vee \neg x_{112}$$

$$2. \neg(x_{111} \wedge x_{113}) = \neg x_{111} \vee \neg x_{113}$$

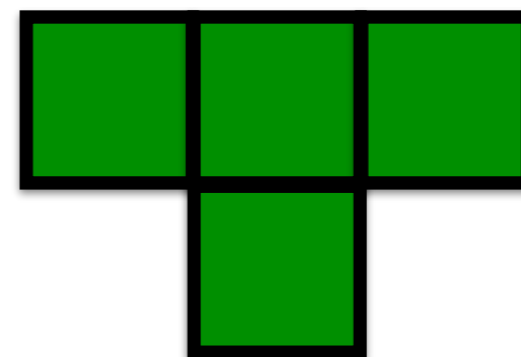
$$3. \neg(x_{112} \wedge x_{113}) = \neg x_{112} \vee \neg x_{113}$$

$$\bigwedge_{(i,j)} \bigwedge_{n \neq m} \neg(x_{ijn} \wedge x_{ijm}) = \bigwedge_{(i,j)} \bigwedge_{n \neq m} (\neg x_{ijn} \vee \neg x_{ijm})$$

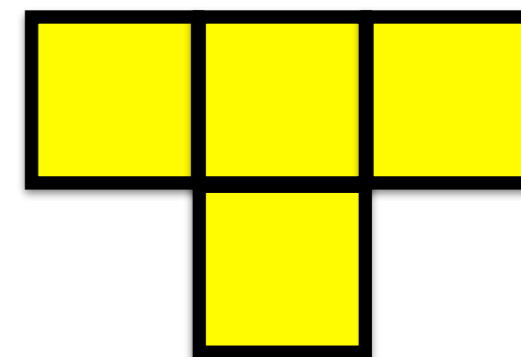
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)



1



2



3

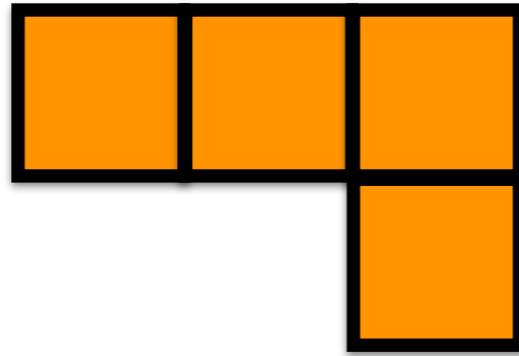
Third condition: “every piece is used at least once.”

One possible encoding: for a piece with n squares, specify that every combination of $n + 1$ grids being assigned with that symbol evaluates to *false*. But this explodes in size.

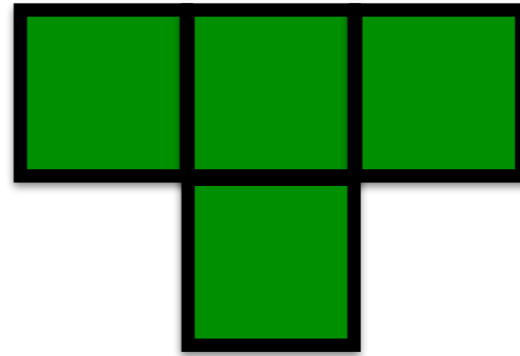
Since the whole grids will be filled when all pieces are used just once, we can restate the condition as the following: for each piece, at least one of the grids is assigned to it. For example, $x_{111} \vee x_{121} \vee x_{131} \vee \dots \vee x_{341}$. To combine for n pieces:

$$\bigwedge_n \bigvee_{(i,j)} x_{ijn}$$

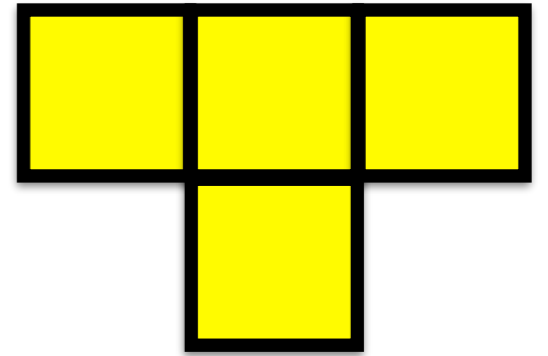
(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)



1



2



3

Fourth condition: “pieces cannot be broken”.

That is, for piece 1, either $x_{111} \wedge x_{121} \wedge x_{131} \wedge x_{231}$ or
 $x_{121} \wedge x_{131} \wedge x_{141} \wedge x_{241}$ or
 $x_{211} \wedge x_{221} \wedge x_{231} \wedge x_{331}$ or ...

Unfortunately, this part is NOT in CNF.

So I cheated a bit; instead of `minisat`, I turned to `z3`.

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)

3	2	2	2
3	3	2	1
3	1	1	1

```

(define-fun x20 () Bool true) 2,3,2
(define-fun x5  () Bool true) 1,2,2
(define-fun x8  () Bool true) 1,3,2
(define-fun x11 () Bool true) 1,4,2

(define-fun x3  () Bool true) 1,1,3
(define-fun x15 () Bool true) 2,1,3
(define-fun x27 () Bool true) 3,1,3
(define-fun x18 () Bool true) 2,2,3

(define-fun x22 () Bool true) 2,4,1
(define-fun x28 () Bool true) 3,2,1
(define-fun x31 () Bool true) 3,3,1
(define-fun x34 () Bool true) 3,4,1

```

<https://github.com/Z3Prover/z3>

<http://rise4fun.com/z3/tutorial>